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General bound-state structure of the massive Schwinger model

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Abstract

Within the Euclidean path integral and mass perturbation theory we derive, from the Dyson-Schwinger equations of the massive Schwinger model, a general formula that incorporates, for sufficiently small fermion mass, all the bound-state mass poles of the massive Schwinger model. As an illustration we perturbatively compute the masses of the three lowest bound states.

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1 Introduction

The massless Schwinger model - which is two-dimensional QED with one massless fermion - is wellknown to be exactly soluble ([1] - [4], [8] - [10]), and its solution may be used as a starting point for a (fermion) mass perturbation theory of the massive Schwinger model ([17], [18], [15], [19]). In both models instantons and a nontrivial vacuum structure (θ-vacuum) are present ([7] - [12]). The spectrum of the massless model consists of one *free*, massive boson with Schwinger mass $\mu_0^2 = \frac{e^2}{\pi}$ (fermion-antifermion bound state [5], [6]) and trivial higher states consisting of n free Schwinger bosons.

For the massive model these higher states turn into n-boson bound states. Their masses, in principle, could be computed, using mass perturbation theory, by evaluating the mass poles of the corresponding n-point functions. Here we will adapt a slightly different method. By exploiting the Dyson-Schwinger equations of the model we will find that all bound-state mass poles are contained within one formula. From this we will compute the masses of the three lowest bound states perturbatively.

All computations are performed within the Euclidean path integral formalism and are done for general vacuum angle θ. This latter fact causes some minor complications, because for $\theta \neq 0$ parity is no longer conserved and, as a consequence, the mass pole equations will turn into matrix equations.

2 Massless Schwinger model

Before starting the actual computations, we need some formulae from the massless Schwinger model. Indeed, the vacuum functional and Green functions of the massive Schwinger model in mass perturbation theory may be traced back to space-time integrations of VEVs of the massless model,

$$Z(m,\theta) = \sum_{k=-\infty}^{\infty} e^{ik\theta} Z_k(m) \tag{1}$$

(k ... instanton number) where

$$Z_k(m) = N \int D\bar{\Psi} D\Psi DA_k^\mu \sum_{n=0}^{\infty} \frac{m^n}{n!} \prod_{i=1}^{n} \int dx_i \bar{\Psi}(x_i)\Psi(x_i) \cdot$$

$$\cdot\, e^{\int dx \left[\bar{\Psi}(i\partial\!\!\!/ - eA\!\!\!/_k)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]} \tag{2}$$

and the mass perturbation expansion is yet performed. A general VEV of the massive model is given by

$$\langle \hat{O} \rangle_m = \frac{1}{Z(m,\theta)} \langle \hat{O} \sum_{n=0}^{\infty} \frac{m^n}{n!} \prod_{i=1}^{n} \int dx_i \bar{\Psi}(x_i)\Psi(x_i) \rangle_0 . \tag{3}$$

For these expressions we need (pseudo-) scalar and vectorial VEVs of the massless model. It is useful to rewrite the scalar densities in terms of chiral ones, $S(x) = S_+(x) + S_-(x)$, $S_\pm \equiv \bar{\Psi} P_\pm \Psi$, because for VEVs of chiral densities only a definite instanton sector $k = n_+ - n_-$ contributes,

$$\langle S_{H_1}(x_1)\cdots S_{H_n}(x_n)\rangle_0 = e^{ik\theta}\Big(\frac{\Sigma}{2}\Big)^n \exp\Big[\sum_{i<j}(-)^{\sigma_i\sigma_j}4\pi D_{\mu_0}(x_i-x_j)\Big] \tag{4}$$

(see e.g. [7]–[9], [20], [17] for the computation) where $\sigma_i=\pm1$ for $H_i=\pm$, D_{μ_0} is the massive scalar propagator,

$$D_{\mu_0}(x) = -\frac{1}{2\pi}K_0(\mu_0|x|), \quad \widetilde{D}_{\mu_0}(p) = \frac{-1}{p^2+\mu_0^2}, \tag{5}$$

($K_0\ldots$ McDonald function) and Σ is the fermion condensate of the massless Schwinger model,

$$\Sigma = \langle\bar\Psi\Psi\rangle_0 = \frac{e^\gamma}{2\pi}\mu_0 \tag{6}$$

($\gamma\ldots$ Euler constant). An inclusion of an arbitrary number of vector currents does not alter the contributing instanton sector and may be computed from the generating functional

$$\langle S_{H_1}(x_1)\cdots S_{H_n}(x_n)\rangle_0[\beta] = e^{ik\theta}\Big(\frac{\Sigma}{2}\Big)^n \exp\Big[\sum_{i<j}(-)^{\sigma_i\sigma_j}4\pi D_{\mu_0}(x_i-x_j)\Big]\cdot$$

$$\cdot\exp\Big[\int dy_1dy_2\beta(y_1)D_{\mu_0}(y_1-y_2)\beta(y_2)+2\sqrt{\pi}\sum_{l=1}^{n}(-)^{\sigma_l}\int dy\beta(y)D_{\mu_0}(y-x_l)\Big]. \tag{7}$$

More precisely, (7) generates all VEVs of n chiral densities and an arbitrary number of Schwinger bosons ϕ, where ϕ is related to the vector current via

$$J_\mu = \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial^\nu\phi. \tag{8}$$

(7) may be found by the inclusion of a vector current source into the path integral quantization and was explicitly computed in [21].

3 The bound-state mass poles

First we have to fix some notation for later convenience:

$$E_\pm(x) := e^{\pm4\pi D_{\mu_0}(x)}-1$$

$$E_\pm^{(n)}(x) := e^{\pm4\pi D_{\mu_0}(x)}-\sum_{l=0}^{n}\frac{1}{l!}(\pm4\pi D_{\mu_0}(x))^l$$

$$\widetilde{E}_\pm^{(n)}(p) = \int d^2x e^{ipx}E_\pm^{(n)}(x) \quad , \quad E_\pm^{(n)} := \widetilde{E}_\pm^{(n)}(0). \tag{9}$$

We will use the following Feynman rules:

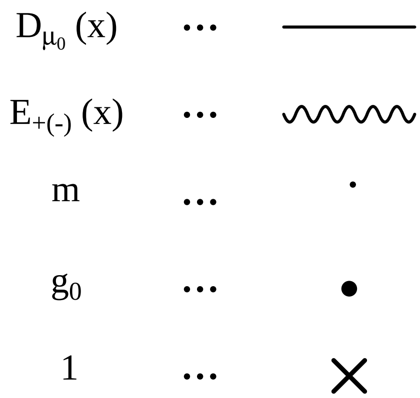

Fig. 1

where m and $g_0 = m\langle S(x)\rangle_m$ are the bare and renormalized coupling, respectively. In the sequel all VEVs are with respect to the massive model, therefore we will omit the subscript m.

We will discuss the special case $\theta = 0$ first, because it is easier and may be represented by simple graphical computations. Later we generalize to arbitrary θ.

On fermionic bilinears there hold two equations of motion, namely the Maxwell equation and the anomaly equation. Eliminating the field strength one arrives at

$$(\Box_x - \mu_0^2)\phi(x) = 2\sqrt{\pi} m P(x) \quad , \quad P = S_+ - S_- \tag{10}$$

where the Schwinger boson ϕ is related to the vector current like in (8). Introducing the abbreviation

$$M_x := \Box_x - \mu_0^2 \tag{11}$$

one may derive Dyson-Schwinger equations like e.g. for the two-point function,

$$\begin{aligned} M_{y_1} M_{y_2} \langle \phi(y_1)\phi(y_2)\rangle = M_{y_1}\delta(y_1 - y_2) + \\ 4\pi g_0 \delta(y_1 - y_2) + 4\pi g_0^2 \langle P(y_1)P(y_2)\rangle, \end{aligned} \tag{12}$$

where it is understood that external sources couple with coupling constant 1. The validity of equation (12) is most easily seen in a graphical representation for the two-point function,

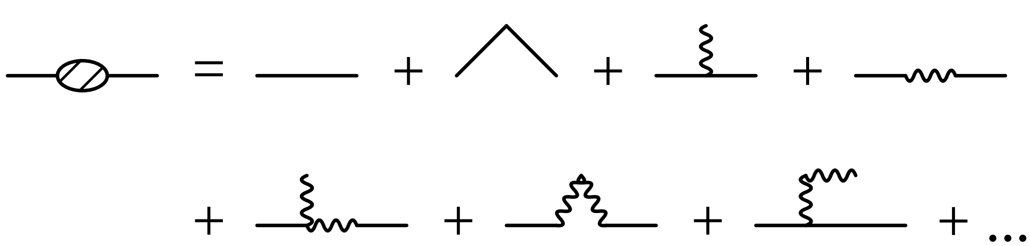

Fig. 2

Here all graphs where the two boson lines meet on one point contribute to the renormalized coupling,

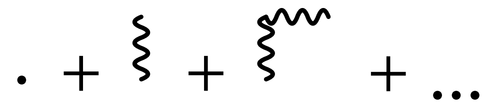

Fig. 3

$$g_0 \equiv m\langle S\rangle = m\Sigma + \frac{1}{2}(m\Sigma)^2(E_+ + E_-) + \dots, \tag{13}$$

see [17], [18] for computational details. All internal vertices are renormalized by the same coupling g_0, and the remaining $\langle P(y_1)P(y_2)\rangle$ part may be written like

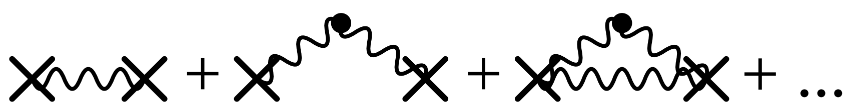

Fig. 4

The $\langle P(y_1)P(y_2)\rangle$ propagator includes, even in least order, an arbitrary number of bosons propagating from y_1 to y_2, which will be essential in the following.

Similar Dyson-Schwinger equations may be derived for higher n-point functions, e.g. for the four-point function

$$\begin{gathered}
M_{y_1}M_{y_2}M_{y_3}M_{y_4}\langle\phi(y_1)\phi(y_2)\phi(y_3)\phi(y_4)\rangle = \\
M_{y_1}M_{y_2}\langle\phi(y_1)\phi(y_2)\rangle M_{y_3}M_{y_4}\langle\phi(y_3)\phi(y_4)\rangle + \text{ perm. } + \\
16\pi^2 g_0\delta(y_1-y_2)\delta(y_1-y_3)\delta(y_1-y_4) + \\
16\pi^2 g_0^2\delta(y_1-y_2)\delta(y_3-y_4)\langle S(y_1)S(y_3)\rangle + \text{ perm. } + \\
16\pi^2 g_0^3\delta(y_1-y_2)\langle S(y_1)P(y_3)P(y_4)\rangle + \text{ perm. } + \\
16\pi^2 g_0^4\langle P(y_1)P(y_2)P(y_3)P(y_4)\rangle
\end{gathered} \tag{14}$$

and analogously for higher n-point functions.

The essential point is that in all these Dyson-Schwinger equations there occurs an identical term that will be responsible for the bound-state formation for sufficiently small fermion mass. In momentum space this term reads

$$c(g_0 + g_0^2\langle\widetilde{PP}\rangle(p)) \tag{15}$$

for odd bound states ($c = 4\pi, 16\pi^2, \dots$) and

$$c(g_0 + g_0^2\langle\widetilde{SS}\rangle(p)) \tag{16}$$

for even bound states.

Now both terms (15), (16) may be inverted via the geometric series formula, e.g.

$$g_0(1 + g_0\langle\widetilde{PP}\rangle(p)) = \frac{g_0}{1 - g_0\langle\widetilde{PP}\rangle_{\text{n.f.}}(p)} \tag{17}$$

where n.f. stands for nonfactorizable and means that graphs contributing to $\langle\widetilde{PP}\rangle_{\text{n.f.}}$ may not be factorized in momentum space. Its graphical representation looks like

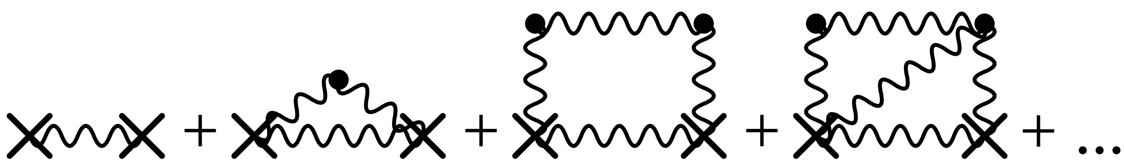

Fig. 5

In (17) the small g_0 has to be compensated by a large contribution from $\langle\widetilde{PP}\rangle_{\text{n.f.}}$ in order to give rise to a mass pole. The first term in Fig. 5 is $E_\pm(x) = e^{\pm 4\pi D_{\mu_0}(x)} - 1$ and contains an arbitrary number of massive propagators D_{μ_0}. Now precisely $(D_{\mu_0}(x))^n$ has a threshold singularity in momentum space at $p^2 = (n\mu_0)^2$, therefore (17) may have mass poles near $p^2 = (n\mu_0)^2$ for $n \in \mathbf{N}$. More precisely, rewriting (for $\theta = 0$) $\langle PP\rangle = 2\langle S_+S_+\rangle - 2\langle S_+S_-\rangle$ (and with a + for $\langle SS\rangle$), and using

$$\langle S_+(x)S_+(0)\rangle = \left(\frac{\Sigma}{2}\right)^2 e^{+4\pi D_{\mu_0}(x)} \quad , \quad \langle S_+(x)S_-(0)\rangle = \left(\frac{\Sigma}{2}\right)^2 e^{-4\pi D_{\mu_0}(x)} \tag{18}$$

we find that $\langle\widetilde{PP}\rangle$ may cause mass poles for *odd* n whereas $\langle\widetilde{SS}\rangle$ may cause mass poles for *even* n, as it has to be. For all mass poles only the terms (15), (16) may balance the pole equation for sufficiently small g_0, therefore it is enough to consider them.

To get more insight we next have to rewrite $\langle\widetilde{PP}\rangle_{\text{n.f.}}$ (Fig. 5) in terms of internal Schwinger bosons (we ignore constants)

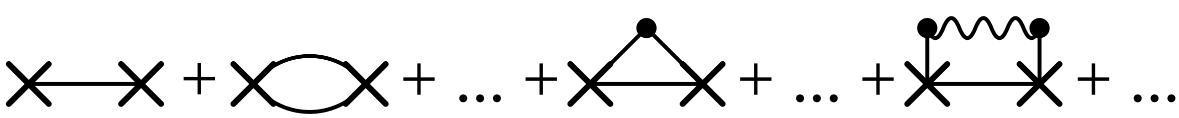

Fig. 6

We find that the one-boson propagator acquires *no* corrections, whereas all terms with more than one boson propagating from y_1 to y_2 have corrections along the boson lines. Consequently, one can compute the lowest pole mass (the Schwinger boson mass) without the need to know it.

On the other hand, for the computation of higher bound-state masses, one has to take into account the corrections, i.e. insert the exact Schwinger mass (the lowest pole mass). The reason is that the mass corrections for the bosons just shift the position of the threshold singularity and are therefore important in lowest order. There are other corrections present, too (internal boson interactions), however, they are unimportant in lowest order.

This result is very plausible physically: the higher bound states should consist of *physical* Schwinger bosons with their physical masses μ (not the bare masses μ_0).

All these features remain true for general θ, only the pole mass formula itself is slightly more complicated and shall be derived next.

4 General θ case

For $\theta \neq 0$ the renormalized coupling is complex,

$$g = m\langle S_+\rangle \quad , \quad g^* = m\langle S_-\rangle \tag{19}$$

and the Dyson-Schwinger equations are slightly changed, too. E.g. the two-point function obeys

$$M_{y_1}M_{y_2}\langle\phi(y_1)\phi(y_2)\rangle = M_{y_1}\delta(y_1-y_2) + (g+g^*)\delta(y_1-y_2)+$$
$$g^2\langle S_+(y_1)S_+(y_2)\rangle + (g^*)^2\langle S_-(y_1)S_-(y_2)\rangle - 2gg^*\langle S_+(y_1)S_-(y_2)\rangle. \tag{20}$$

The interesting function that gives rise to the mass poles is

$$g(1+g\langle\widetilde{S_+S_+}\rangle(p) - g^*\langle\widetilde{S_+S_-}\rangle(p)) + g^*(1+g^*\langle\widetilde{S_-S_-}\rangle(p) - g\langle\widetilde{S_+S_-}\rangle(p)) \tag{21}$$

for odd bound states and with only plus signs for even bound states. Because parity is no longer conserved the P and S components mix and the geometric series formula (17) generalizes to a matrix equation.

Introducing the abbreviations

$$A := 1 + g\langle\widetilde{S_+S_+}\rangle(p) - g^*\langle\widetilde{S_+S_-}\rangle(p)$$
$$\alpha := g\langle\widetilde{S_+S_+}\rangle_{\text{n.f.}}(p) \quad , \quad \beta := g\langle\widetilde{S_+S_-}\rangle_{\text{n.f.}}(p) \tag{22}$$

the equation reads

$$\begin{aligned} A &= 1 + \alpha A - \beta^* A^* \\ A^* &= 1 + \alpha^* A^* - \beta A \end{aligned} \tag{23}$$

and has the solution

$$A = \frac{1-\alpha^*-\beta^*}{1-\alpha-\alpha^*+\alpha\alpha^*-\beta\beta^*}. \tag{24}$$

Equation (23) may be checked by a careful investigation of the perturbative expansion for $\langle P(y_1)P(y_2)\rangle$.

For even bound states the solution may be found from (24) by the substitution $\beta \to -\beta$. Therefore, all solutions have the same denominator, and the zeros of this denominator are the pole masses of all bound states of the massive Schwinger model.

Explicitly the pole-mass equation reads

$$(1-g\langle\widetilde{S_+S_+}\rangle_{\text{n.f.}}(p))(1-g^*\langle\widetilde{S_+S_+}\rangle^*_{\text{n.f.}}(p)) = gg^*(\langle\widetilde{S_+S_-}\rangle_{\text{n.f.}}(p))^2 \tag{25}$$

where

$$\begin{aligned} g &= m\frac{\Sigma}{2}e^{i\theta} + m^2\Big(\frac{\Sigma}{2}\Big)^2(E_+e^{2i\theta}+E_-) + o(m^3) \\ &=: g_1 + g_2 + o(m^3) \end{aligned} \tag{26}$$

$$\langle\widetilde{S_+S_+}\rangle_{\text{n.f.}}(p) = \widetilde{E}_+(p) + o(m) \quad , \quad \langle\widetilde{S_+S_-}\rangle_{\text{n.f.}}(p) = \widetilde{E}_-(p) + o(m). \tag{27}$$

Of course, for $\theta = 0$ (real A, α, β), one recovers the equations (15), (16).

5 Explicit mass computations

For a computation of the Schwinger mass up to second order we rewrite (25) like

$$(1-g\widetilde{E}_+(p))(1-g^*\widetilde{E}_+(p)) = gg^*\widetilde{E}_-^2(p) \tag{28}$$

and separate the one-boson contribution

$$\widetilde{E}_\pm(p) = -\frac{\pm 4\pi}{p^2+\mu_0^2} + \widetilde{E}_\pm^{(1)}(p) \tag{29}$$

leading to

$$-p^2-\mu_0^2 = 8\pi\text{Re}\, g_1 - 2\text{Re}\, g_1\widetilde{E}_+^{(1)}(p)(p^2+\mu_0^2) + 8\pi\text{Re}\, g_2 + 8\pi g_1 g_1^*(\widetilde{E}_+^{(1)}(p) - \widetilde{E}_-^{(1)}(p)) \tag{30}$$

with the solution

$$-p^2 = \mu_0^2(1+4\pi\frac{\Sigma m}{\mu_0^2}\cos\theta) \tag{31}$$

in first order and

$$-p^2 = \mu_0^2\Big[1+4\pi\frac{\Sigma m}{\mu_0^2}\cos\theta + 2\pi\frac{m^2\Sigma^2}{\mu_0^4}\Big((E_+ + \widetilde{E}_+^{(1)}(1))\cos 2\theta + E_- - \widetilde{E}_+^{(1)}(1)\Big)\Big] \tag{32}$$

in second order. Here we rescaled $p \to p' = \frac{p}{\mu_0}$ and used $\widetilde{E}_\pm^{(1)}(p') = \widetilde{E}_\pm^{(1)}(1) + o(m)$ in the last step. This result precisely conicides with the result obtained by a direct perturbative computation ([18]).

In order to compute the two-boson bound state we have to separate the two-boson part of $\widetilde{E}_+$ in (28). In lowest order we find

$$1 = \frac{1}{2!}(g_1+g_1^*)16\pi^2(\widetilde{D_\mu^2})(p) \tag{33}$$

where now μ is the *physical* Schwinger mass (32) including fermion mass corrections. Using

$$(\widetilde{D_\mu^2})(p) = \frac{1}{4\pi(-p^2)}\frac{1}{\sqrt{\frac{4\mu^2}{-p^2}-1}}\arctan\frac{1}{\sqrt{\frac{4\mu^2}{-p^2}-1}} \tag{34}$$

(see e.g. [22] for a computation) and remembering that $4\mu^2 - (-p^2)$ is a very small number (that is *positive* for a bound state) we may set $\frac{1}{-p^2} \simeq \frac{1}{4\mu^2}$, $\arctan(\cdots) \simeq \frac{\pi}{2}$ and get

$$-p^2 \simeq 4\mu^2 (1 - \frac{\pi^4}{16} \frac{m^2 \Sigma^2}{\mu^4} \cos^2\theta) \tag{35}$$

which is of second order in m. Again, this result coincides with the one from a direct perturbative calculation ([22]).

6 The three-boson bound-state mass

For the three-boson bound-state mass we have to separate the three-boson part in (28) and find, in lowest order

$$1 = \frac{1}{3!} m\Sigma \cos\theta \cdot 64\pi^3 (\widetilde{D^3_\mu})(p) \tag{36}$$

or, after a rescaling $p \to \frac{p}{\mu}$ to dimensionless momenta

$$1 = \frac{64\pi^3}{6} \frac{m\Sigma}{\mu^2} \cos\theta\, (\widetilde{D^3_\mu})(p). \tag{37}$$

$(\widetilde{D^3_\mu})(p)$ is given by the graph (where we introduce positive squared momentum $Q^2 = -p^2 > 0$)

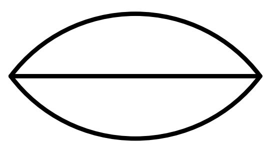

Fig. 7

$$- \int \frac{d^2 q_1 d^2 q_2}{(2\pi)^4} \frac{1}{(p+q_1+q_2)^2+1} \frac{1}{q_1^2+1} \frac{1}{q_2^2+1} =$$

$$-2 \int_0^1 dx \int_0^x dy \int \frac{d^2 q_1 d^2 q_2}{(2\pi)^4} \frac{1}{\left[q_1^2 + 1 + (q_2^2 - q_1^2)x + ((p+q_1+q_2)^2 - q_2^2)y\right]^3} =$$

$$\int \frac{dx}{(4\pi)^2} \int_0^x \frac{dy}{Q^2(xy - x^2 y - y^2 + xy^2) - x + x^2 - xy + y^2} =$$

$$\int \frac{dx}{8\pi^2(1 - Q^2(1-x))} \int_0^{\frac{x}{2}} \frac{dz}{z^2 + T^2(Q^2, x)} =$$

$$\int_0^1 \frac{dx}{8\pi^2(1 - Q^2(1-x))} \frac{1}{T(Q^2,x)} \arctan \frac{x}{2T(Q^2,x)}, \tag{38}$$

where

$$T^2(Q^2, x) = \frac{x^2 - Q^2 x^2 (1-x) + 4x(1-x)}{4(Q^2(1-x) - 1)}. \tag{39}$$

The numerator of T^2 has a double zero at $Q^2 = 9$:

$$9x(x - \frac{2}{3})^2. \tag{40}$$

This double zero is in the integration range of x and is precisely the threshold singularity. Setting

$$Q^2 =: 9(1 - \epsilon) \tag{41}$$

in the numerator of T^2 in the factor $\frac{1}{T}$, and $Q^2 = 9$ everywhere else, where it is safe, one arrives at:

$$\frac{1}{12\pi^2} \int_0^1 \frac{dx}{\sqrt{|9x-8|}} \frac{\arctan \frac{\sqrt{x|9x-8|}}{3(x-\frac{2}{3})}}{\sqrt{(x-\frac{2}{3})^2 x + \epsilon x^2(1-x)}} =: I(\epsilon). \tag{42}$$

The mass-pole equation reads ($\frac{m\Sigma}{\mu^2} \cos\theta \equiv \frac{1}{\mu^2} 2\mathrm{Re}\, g_1 =: \alpha$)

$$1 = \frac{64\pi^3}{6} \alpha I(\epsilon) \tag{43}$$

and must be evaluated numerically. It gives rise to an extremely tiny mass correction ϵ. For sufficiently small α it is very well saturated by

$$\epsilon(\alpha) \simeq 0.777 \exp(-\frac{0.263}{\alpha}) \tag{44}$$

and is therefore smaller than polynomial in α. (I checked the numerical formula (44) for $30 < \frac{1}{\alpha} < 1000$, corresponding to $10^{-3} < \epsilon < 10^{-100}$, but I am convinced that it remains true for even larger $\frac{1}{\alpha}$; however, there the numerical integration is quite difficult because of the pole in (42).)

A more accurate mass-pole equation would include some additional contributions:

$$1 = \alpha(\frac{64\pi^3}{6} I(\epsilon) + f(\epsilon)) \tag{45}$$

where $f(\epsilon)$ is some function that is finite for $\epsilon \to 0$. But for α sufficiently small it remains true that an extremely tiny value of ϵ suffices to saturate the mass-pole equation, whatever the value of $f(\epsilon)$ is.

We conclude that the three-boson bound state mass is nearly entirely given by three times the Schwinger boson mass,

$$M_3^2 \simeq 9\mu^2, \tag{46}$$

or, differently stated, that the binding of three bosons is extremely weak.

This result enables us to add a short remark on a result that was obtained in [15]. There it was argued that, for $\theta = 0$, the three-boson bound state should be stable because a decay into two Schwinger bosons is forbidden by parity conservation. However, because of (46) it holds that $M_3 > M_2 + \mu$, therefore there should be a small probability for the three-boson bound state to decay into one two-boson bound state and one Schwinger boson.

7 Summary

As claimed, we arrived at our aim to derive one formula for all bound-state masses of the massive Schwinger model, at least for sufficiently small fermion mass. Of course, if we were able to exactly solve the model, it would not at all be surprizing to find all mass poles within one Green function. The interesting point is that we could reach this aim by the use of the Dyson-Schwinger equations and by a specific partial resummation of the perturbation series.

We found that the two lowest states acquire noticeable corrections, whereas the binding energy of the three-boson bound state is extremely tiny. It is plausible to conjecture that the binding energies of higher bound states remain very tiny.

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